From Montague Semantics to MTT-semantics: A Meaningful Comparison

Lecture 4: Case study on copredication

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Copredication

- More than one predicate, representing a verb or an adjective and requiring different types of arguments

(1) John picked up and mastered the book

(2) The lunch was delicious but took forever

- *Picked up* seems to require a physical object while *mastered* an informational one. *The book* is thus interpreted as involving both a physical and an informational aspect
- Similarly, *lunch* appears to involve both an a *event* as well as a *food* aspect
- Originally (at least to a great extent), discussed in Pustejovksy (1995)
    - It has given rise to a number of approaches since then (Cooper 2008, Asher 2011, Luo 2010, Chatzikyriakidis and Luo 2015, Gotham 2014, 2016 among others)

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Copredication and Individuation criteria

- Copredication interacts in interesting ways with individuation. Consider the following:

  (3) John picked up and mastered three books

- What the above sentence means is that John picked up three physical objects and mastered three informational ones
  - It cannot mean that John picked up three physical objects but mastered one informational object (e.g. a case where John picked up three copies of the same book) or vice versa
  - Accounts of copredication should be able to correctly capture the individuation facts
  - Our account of individuation should be elaborate enough in order to capture this data

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The earlier accounts in MTT semantics: Luo's 2010 account

- The account for *dot-types* as proposed in [Luo(2010), Xue and Luo(2012)] proceeds as follows (we use the example of physical and informational objects):
  - $\textsc{Phy}$ and $\textsc{Info}$ are the types of physical objects and informational objects
  - $\textsc{Phy} \bullet \textsc{Info}$ is the dot-type of objects having both a $\textsc{Phy}$ and an $\textsc{Info}$ aspect
  - The *dot-type* is a subtype of its constituent types
    $\textsc{Phy} \bullet \textsc{Info} < \textsc{Phy}$
    $\textsc{Phy} \bullet \textsc{Info} < \textsc{Info}$
  - The two aspects of *book* are then represented as: $Book < \textsc{Phy} \bullet \textsc{Info}$

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Luo's account of dot types- An example

- Let us consider the following example:

(4) John picked up and mastered the book.

- The verbs are given the following typings:

$$
\begin{aligned}
[\![\textit{pick up}]\!] &: [\![\textit{human}]\!] \rightarrow \textsc{Phy} \rightarrow \textit{Prop} \\
[\![\textit{master}]\!] &: [\![\textit{human}]\!] \rightarrow \textsc{Info} \rightarrow \textit{Prop}
\end{aligned}
$$

- Due to contravariance of function types (and the declared subtyping relations of course) we have:

$$
\begin{aligned}
[\![\textit{pick up}]\!] \quad &: \quad [\![\textit{human}]\!] \rightarrow \textsc{Phy} \rightarrow \textit{Prop} \\
&< \quad [\![\textit{human}]\!] \rightarrow \textsc{Phy} \bullet \textsc{Info} \rightarrow \textit{Prop} \\
&< \quad [\![\textit{human}]\!] \rightarrow [\![\textit{book}]\!] \rightarrow \textit{Prop}
\end{aligned}
$$

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Luo's account of dot types- An example

- Continued

$$\begin{array}{rcl} [\![master]\!] & : & [\![human]\!] \to \textsc{Info} \to Prop \\ & < & [\![human]\!] \to \textsc{Phy} \bullet \textsc{Info} \to Prop \\ & < & [\![human]\!] \to [\![book]\!] \to Prop \end{array}$$

- Therefore, $[\![pick\ up]\!]$ and $[\![master]\!]$ can both be used in a context where terms of type $[\![human]\!] \to [\![book]\!] \to Prop$ are required
- See the paper for the formal details and introduction/elimination rules for dot-types

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Chatzikyriakidis and Luo 2015 account

- The authors there try to show that the account as is, will give the correct identity criteria
- Formalization in the proof assistant Coq
  - There is no universe construction in Coq, so the universe is CN is taken to be Set
  - *Man* and *Human* are declared as types. The third line involves declaring subtyping relations
  - The quantifier *three* is given a definition according to which three elements different to each other are hold for a property P
  - The rest of the code follows the analysis in Luo (2011)

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Chatzikyriakidis and Luo 2015 account

```
Definition CN:= Set.
Parameter Man Human: CN. Parameter John: Man.
Axiom mh: Man->Human. Coercion mh: Man>->Human.
Parameter Phy Info: CN.
Record PhyInfo: CN:= mkPhyInfo{phy:>Phy;info:>Info}.
Definition Three:= fun(A: CN)(P: A->Prop)=>exists x: A,P x /\
(exists y: A,P y/\(exists z: A,P z /\ x<>y /\ y<>z /\ x<>z)).
```

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Chatzikyriakidis and Luo 2015

- We need to be able to capture the following inferences:
  1. John picked up and mastered three books $\Rightarrow$ John picked up three books and mastered three books
  2. John picked up and mastered three books $\Rightarrow$ John picked up three physical objects and mastered three informational objects
- We formulated the desired inferences as Coq theorems
- We attempt to prove them using Coq's proof tactics
  - 1. is easilyproven
  - 2. does not go through

Chatzikyriakidis and Luo 2015

- The authors (well, us!) introduce axioms for equality under subtyping:

  In general, when $X <_c Y$*, we do not have* $x \neq_X y \implies (x \neq_Y y)$ *unless* c *is injective. For the atomic types like Book and* $\textsc{Phy}$*, the equality on a subtype coincides with that of the supertype and so we can axiomatically assume this.* (Chatzikyriakidis and Luo, 2015)

- The following two axioms are encoded in Coq in this respect

```
Variable PHY:forall x:Book, forall y:Book,
not(x=y:>Book)-> not(x=y:>Phy).
Variable INFO:forall x:Book, forall y:Book,
not(x=y:>Book)-> not(x=y:>Info).
```

- These axioms help in proving the examples involving *three*. But at a cost!

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Chatzikyriakidis and Luo 2015

- The account overgenerates
  - Examples like the following can also be proven:

    (5) John picked up three books and John mastered every book $\Rightarrow$ John mastered three books.

  - Thanks to Matthew Gotham who pointed this out to us!
- In general: this is not a principled solution but a hack

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What is the problem?

- The previous accounts relied on the implicit idea that every type defines its own equivalence relation
  - CNs are basically setoids (as e.g. argued in Luo 2012). Thus, the interpretation of a CN is not just a type, rather a type associated with an identity criterion for that CN)
  - A type plus an equivalence relation on the type
    - An IC if you wish
- However, this has not been explicitly specified in the previous accounts of individuation
- Individuation in copredication is a clear case where this idea needs to get explicit
  - This is what we are going to do here

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Criteria of Identity/Individuation: CNs as Setoids

- Individuation is the process by which objects in a particular collection are distinguished from one another
  - Provides us with means to count and a sameness criterion
- In linguistic semantics, individuation is related to the idea that a CN may have its own identity criterion for individuation [Geach(1962)]
- Mathematically, the association of an equivalence relation (the identity criterion) CNs
  - In constructive mathematics, a set or a type is indeed a collection of objects together with an equivalence relation that serves as identity criterion of that collection

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CNs as Setoids

- CNs are not just types
  - Types plus an identity criterion for that specific CN

    (6) $(A, =)$

    where A is a type and $=: A \to A \to \mathit{Prop}$ is an equivalence relation over A
  - The difference between CNs-as-Types and CNs-as-Setoids

    (7) $[\mathit{human}] = \mathit{Human} : \mathit{Type}$ (CNs-as-types view)

    (8) $[\mathit{human}] = (\mathit{Human}, =_h)$ (CNs-as-Setoids view)

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CNs as Setoids

- Consider the following examples and their semantic interpretations:

(9) Three men talk.

(10) Three humans talk.

(11) $\exists x, y, z : Man.\ x \neq_M y \ \wedge y \neq_M z \ \wedge x \neq_M z \ \wedge talk(x) \ \wedge talk(y) \ \wedge talk(z)$

(12) $\exists x, y, z : Human.\ x \neq_H y \ \wedge y \neq_H z \ \wedge x \neq_H z \ \wedge talk(x) \ \wedge talk(y) \ \wedge talk(z)$

where $\mathrm{M{\scriptsize AN}} = (Man, =_M)$ and $\mathrm{H{\scriptsize UMAN}} = (Human, =_H)$ are setoids and the identity criterion for men and that for humans are used to express that x, y and z are distinct from each other.

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CNs as Setoids

- Necessary to consider the individuation criteria explicitly by using the identity criteria $=_M$ and $=_H$
- The relationship between the $\textsc{Man}$ and $\textsc{Human}$ is one where the first inherits the IC from the second

(13) $(=_M) = (=_H)|_{Man}$

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CNs as Setoids: Subsetoids

- $\mathrm{A} = (A, =_{\mathrm{A}})$ is a $\mathrm{sub\text{-}setoid}$ of $\mathrm{B} = (B, =_{\mathrm{B})}$, notation $\mathrm{A} \sqsubseteq \mathrm{B}$, iff
  - $A \leq B$ and $=_{\mathrm{A}}$ is the same as $(=_{\mathrm{B}})$
- Some examples:

(14) $\textsc{Man} \sqsubseteq \textsc{Human}$

(15) $(RTable, =_t) \sqsubseteq (Table, = t)$
where $RTable$ is: $\Sigma x{:}Table.red(x)$ is the domain of red tables and $=_t$ is the equivalence relation representing the identity criterion for tables

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CNs as Setoids: Subsetoids

- In restricted domains like *Man* or *RTable*, the identity criteria coincide with those in *Human* and *Table*
- For these cases, one can ignore the IC, i.e. one can use the simpler CNs-as-Types approach
  - More sophisticated cases like copredication with quantification however need IC

(16) John picked up and mastered three books.

- Double distinctness

(17) John picked up and mastered three books $\Rightarrow$ John picked up three physical objects and mastered three informational objects

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CNs as Setoids: Copredication

- Let us split the example into its conjuncts

(18) $\mathit{Three}(\mathit{Book}, \textsc{Phy}, \mathit{pick\ up}(j))$.

(19) $\mathit{Three}(\mathit{Book}, \textsc{Info}, \mathit{master}(j))$.

- Note: the CN *book* in 18 refers to a different collection from that referred to by *book* in 19

(20) $\textsc{Book}_1 = (\mathit{Book}, =_p)$

(21) $\textsc{Book}_2 = (\mathit{Book}, =_i)$

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CNs as Setoids: Copredication

- How the identity criterion for *books* is determined
  - why do we use $=_p$ in 18 and $=_i$ in 19?
    - ★ The verb (and its semantics) determines the identity criterion of the object CN.

$$IC^{N,V} \Rightarrow \begin{cases} =_p & \text{if Dom(V)} = \textsc{Phy} \\ =_i & \text{if Dom(V)} = \textsc{Info} \\ ??? & \text{if Dom(V)} = \textsc{Phy} \bullet \textsc{Info} \end{cases}$$

    - ★ In order to deal with the dot-type case, we have to define setoids for dot-types!

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CNs as Setoids: Copredication

- Let $\mathrm{A} = (A, =_{\mathrm{A}})$ and $\mathrm{B} = (B, =_{\mathrm{B}})$ be setoids. Then, the *dot-setoid* $\mathrm{A} \bullet \mathrm{B}$ is defined as follows:
  - $\mathrm{A} \bullet \mathrm{B} = (A \bullet B,\ =_{\mathrm{A} \bullet \mathrm{B}})$
    where $\langle a_1, b_1 \rangle =_{\mathrm{A} \bullet \mathrm{B}} \langle a_2, b_2 \rangle$ if, and only if, $(a_1 =_{\mathrm{A} a_2}) \vee (b_1 =_{\mathrm{B}} b_2)$.

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CNs as Setoids: Copredication

- The semantics for *three*
  Let A be a type and $\mathrm{B} = (B, =_{\mathrm{B}})$ a setoid such that $A \leq B$, and $P : B \rightarrow Prop$ a predicate over B:
- $Three(A, \mathrm{B}, P) = \exists x, y, z : A.\ D[\mathrm{B}](x, y, z)\ \wedge P(x)\ \wedge P(y)\ \wedge P(z)$.
  where $D[\mathrm{B}](x, y, z) = x \neq_{\mathrm{B}} y\ \wedge y \neq_{\mathrm{B}} z\ \wedge x \neq_{\mathrm{B}} z$.

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CNs as Setoids: Copredication

- With these definitions, the desired semantics of our copredication cases are derived
- $\mathit{Three}(\mathit{Book}, \textsc{Phy} \bullet \textsc{Info}, \mathit{pm}(j))$
- $\exists x, y, z : \mathit{Book}.D[\textsc{Phy}](x, y, z)\ \&\ D[\textsc{Info}](x, y, z)\ \&\ \mathit{pm}(j, x)\ \&\ \mathit{pm}(j, y)\ \&\ \mathit{pm}(j, z)$
    - Note that this is achieved through defining the equivalence relation for dot-types by means of disjunction of both identity criteria and, then, we obtain double distinctness by negating the disjunction.

CNs as Setoids: Copredication

- Verbs plus adjectives in quantified copredication
  - Consider the following example:

    (22) John mastered three heavy books.

  - The interpretation needed here: John mastered three informational objects that are also heavy as physical objects
    - ★ Both the verb and the adjective have a word on the IC
- First step: adjectival modification
  - $HBook = \Sigma(Book, heavy)$ or $\Sigma x{:}Book.heavy(x)$

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CNs as Setoids: Copredication

- The interpretation we get:

  $Three(HBook, \textsc{Phy} \bullet \textsc{Info}, master(j))$

- Expanding:

  $\exists x, y, z : HBook.D[\textsc{Phy}](x, y, z)\ \&\ D[\textsc{Info}](x, y, z)$
  $\&\ master(j, x)\ \&\ master(j, y)\ \&\ master(j, z)$

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Gotham 2016

- If we want to be precise, Gotham's account [Gotham(2016)] is not Montagovian *per se*, since the underlying logic is not set theory but mereology
  - However, mereological accounts within the mainstream Montagovian tradition have been quite common from [Link(1983)] onwards
  - We thus consider such accounts within the Montagovian general tradition

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Gotham 2016: The key components of the account

- Both complex and plural objects exist
- Complex objects are denoted with with the $+$ operator while plural objects with the $\oplus$ operator (with assumes a join semilattice ála [Link(1983)]
  - For example, *book* denotes the set of composite objects p + i, where p is a physical book and i is an informational book
  - Gotham further assumes that any property that holds of p holds of $p + i$, and likewise any property that holds of i also holds of $p + i$.
    - ★ $\forall P.P(p) \rightarrow P(p + i)$ and $\forall P.P(i) \rightarrow P(p + i)$
- Important to note: Plural objects exist in Link's system but not complex objects!

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Gotham 2016: The key components of the account

- Lexical entries are more complex than what is traditionally assumed in the sense that besides deciding extensions, they further specify a distinctness criterion
- First proposal for the type of common nouns: $e \rightarrow (t \times \mathbf{R})$
  - $(t \times R)$ is a product type, with t a truth value and R the type of relations, basically an abbreviation for $e \rightarrow e \rightarrow t$.
- Final proposal: $e \rightarrow (t \times ((e \rightarrow \mathbf{R}) \rightarrow t))$ (abbreviated to $e \rightarrow T$)
  - the second projection π_2 is a set of functions that map the type e argument to a relation of type $e \rightarrow \mathbf{R}$
    - where $R \sqsubseteq R_{cn}$, and R_{cn} is taken to be the individuation relation given by the noun or the predicate.

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Gotham 2016: The key components of the account

- Common nouns involve an individuation relation in their lexical entries.
  - These are equivalences for different objects exist, for example physical, informational etc.
    1. $\mathrm{PHY} = \lambda x, y : e.\mathit{phys\text{-}equiv}(x)(y)$
    2. $\mathrm{INFO} = \lambda x, y : e.\mathit{info\text{-}equivequiv}(x)(y)$
- Lexical entry for *book*:
  $\lambda y : e.^{*}\mathit{book}(y), \lambda f : e \rightarrow R.f(y) \sqsubseteq (\mathrm{PHYS} \sqcup \mathrm{INFO})$

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Gotham 2016: The key components of the account

- A non-compressibility statement denoting that no two members of a plurality stand to a relation R (see [Gotham(2014)] for the formal definition of compressibility)
- A further assumption that verbs also somehow point to the individuation criteria that have to be used
  1. For this, a form of generalized implication is used
- Lastly an Ω operator is used that helps in choosing the individuation criteria for each of the arguments
  - Ω computes the least upper bound of the set of relations R.
    - E.g., in the case of *books* this is $(\mathrm{PHYS} \sqcup \mathrm{INFO})$, for *master* it is (*INFO*) and so on.

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Comparing the Approaches

- Common claim in both: common nouns need to be extended with the addition of individuation criteria
  - Gotham extends the traditional notion of predicates as sets to a considerably more complex type ($e \rightarrow (t \times ((e \rightarrow \mathbf{R}) \rightarrow t)))$
  - We claim that CNs are not just types anymore, but rather setoids with their second component being an equivalence relation on the type $((A, =))$ (Gotham's R)

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Comparing the Approaches

- Deciding the IC for verbs or adjectives
  - Gotham uses the Ω function
  - For our account, the IC criteria for the arguments of the verb are decided by the type of the argument and whether it is a complex object or not

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Comparing the Approaches

- Potential problem?
  - Gotham uses a mereological account to capture the individuation criteria
    - ★ makes an additional crucial, non-trivial assumption: the introduction of the $+$ operator and the axiom $P(a) \Rightarrow P(a+b) \wedge P(b+a)$, for any predicate P
    - ★ Crucial for the account to work
    - ★ Plays the role of the definition we have for dot-setoids.
    - ★ One can question its naturalness, as [Gotham(2016)] seems to be doing:

One can imagine other ways of implementing this mereological approach to , both in terms of what composite objects there are and in terms of what their properties are. That in turn would require a revision to the definition of compressibility in Section 3.1.1. The approach adopted in this article is adequate for criteria of individuation to be determined compositionally, and can be adapted if revisions of this kind turn out to be necessary [GOT 17: fn2]

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Comparing the Approaches

- Our account and introduction of follows the standard way of forming types in MTTs, i.e. via formation/introduction/elimination/computational rules. This seems to be an advantage of our approach
- Most importantly, a more serious, formal flaw
  - Assume the following predicate P:
    $P(p) = \text{true if } p = a + x$, $P(p) = \text{false otherwise}$
  - Then, $P(a)$ is false and $P(a + x)$ is true and thus P does not satisfy Gotham's axiom
  - We do not know how serious of a a flaw this is and whether a trivial fixing is available.

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